

Muskingum-Cunge Flood Routing Procedure in NRCS Hydrologic Models

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ABSTRACT: The Muskingum-Cunge flood routing procedure has been incorporated into the Natural Resources Conservation Service (NRCS) Technical Release 20 (TR-20) hydrologic model. The TR-20 model is an event watershed hydrologic model used to analyze impacts of watershed changes (land use, reservoir construction, channel modification, etc) on volume of runoff and peak discharge. The model consists of various components such as runoff generation, channel and reservoir routing, and floodwater diversion. The revised NRCS Win TR-55 (Urban Hydrology for Small Watersheds) computer program utilizes TR-20 to do its hydrologic computations and thus, routing results will be based on the Muskingum-Cunge procedure. After much research, development, and testing, an original formulation has been adopted. The uniqueness of this formulation lies in the way flood wave celerity is determined for the reach. The flood wave celerity is equal to a factor multiplied by the average velocity. For natural channels it is typically estimated as the slope of a plot of discharge versus flow area relationship. When water exceeds the channel banks and enters the flood plain, the slope of the discharge – flow area curve multiplied by the average flow velocity is not representative of the actual flood wave celerity. An algorithm to compute the factor (denoted **m**) for a rating curve developed from a steady flow water surface profile model is presented.

A series of tests were conducted which compared the results of the Muskingum-Cunge formulation against a solution of the one-dimensional St. Venant equations. The tests were designed to cover a range of non-dimensional and actual conditions to indicate limits of applicability. The solution of the St. Venant equations was considered an accurate solution to the physical problem due to the number of validation studies conducted in both laboratory and natural settings. Routing tests were completed for trapezoidal and natural cross sections. Accuracy of the peak discharge at the end of the reach, the time to peak discharge at the end of the reach, and the correlation of hydrograph shapes are presented.

INTRODUCTION

The Muskingum-Cunge flood routing procedure has been incorporated into the Natural Resources Conservation Service (NRCS) Technical Release 20 (TR-20) hydrologic model. The Muskingum-Cunge method for channel flood routing has been documented in many textbooks, professional papers, and by Dr. Miguel Ponce in two reports delivered by contract to SCS in 1981 and 1983. The Corps of Engineers HEC added it as a flood routing option in the HEC-1 program (1990). Routing tests performed by Younkin and Merkel (1986, 1988) showed increased accuracy, consistency, and range of physical conditions when compared to both the Modified Att-Kin and Convex flood routing methods which had been used previously (NRCS, 1965 and 1983). The Muskingum-Cunge routing procedure was compared to the solution of the dynamic wave equations as formulated by St. Venant. The one-dimensional dynamic wave model was used as a bench mark or “correct” solution to the routing problem. The theoretical

development of the Muskingum-Cunge routing equation is based on the simplification of the convective diffusion equation and is documented in the two Ponce reports. The routing equation is the same as the traditional Muskingum routing method (Linley, Kohler, Paulhus, 1982). The coefficients of the Muskingum-Cunge method are based on data such as cross section and estimated Manning n. The coefficients of the traditional Muskingum method can only be determined from stream gage data. This makes the Muskingum-Cunge method more favorably applied to ungaged streams (which vastly outnumber gaged streams). Since the TR-20 program is being re-written and modularized, it was decided to add a Muskingum-Cunge flood routing technique to replace the Modified Att-Kin method.

The Muskingum-Cunge has two formulations which relate to estimation of routing coefficients and method of mathematical solution. They include:

1. Constant coefficient method. This formulation is based on selection of a reference discharge for which the routing coefficients are computed. The routing equation is then solved based on these coefficients. The reference discharge tested in TR-20 was the peak inflow and average of peak flow and base flow. One of the characteristics of this formulation is that volume of inflow (upstream end of the reach) is always equal to the volume of outflow (downstream end of the reach). This formulation also has an efficient direct solution technique.
2. Variable coefficient method. This formulation is based on the routing coefficients varying by time step to reflect flow characteristics (primarily wave celerity, friction slope, and top width) of the rising and receding flood wave. In reality, flow in the channel travels at a different speed and naturally has different cross section top width than flow in the flood plain. The variable coefficient method reflects these differences and matches the shape of the outflow hydrograph somewhat better than the constant coefficient formulation (when compared to the solution of the dynamic wave equations). The two solution techniques proposed by Ponce (1983) are a three-point direct and four point iterative procedure. The three point direct solution bases the routing coefficients on the three known discharges (inflow at first and second time steps and the outflow at the first time step). The four point iterative procedure bases the routing coefficients on the three known discharges and the estimate of the outflow at the second time step. Since the outflow at the second time step is not known, iterations are needed to converge on the final discharge based on some error tolerance. The four-point variable coefficient solution technique was tested in TR-20. The variable coefficient methods do not always conserve hydrograph volume (volume of inflow is not always equal to volume of outflow). This is the primary disadvantage to variable coefficients.

MUSKINGUM METHOD

The Muskingum channel routing method is based on two equations (Linsley, Kohler, Paulhus, 1982). The first is the continuity equation or conservation of mass.

$$\frac{I_1 + I_2}{2} \Delta t - \frac{O_1 + O_2}{2} \Delta t = S_2 - S_1 \quad \text{equation 1}$$

where: I1 and I2 are inflow discharges at time 1 and time 2, ft³/s
 O1 and O2 are outflow discharges at time 1 and time 2, ft³/s
 Δt = time difference between time 1 and time 2, s
 S1 and S2 are values of reach storage at time 1 and time 2, ft³.

The second equation is a relationship of storage, inflow, and outflow of the reach.

$$S = K \{ X I + (1 - X) O \} \quad \text{equation 2}$$

where : S = reach storage, ft³
 I = inflow discharge, ft³ / s
 O = outflow discharge, ft³/s
 K = storage constant, s
 X = weighting factor, dimensionless.

Combining equations 1 and 2 and simplifying results (Ponce, 1981):

$$O_2 = C_1 I_1 + C_2 I_2 + C_3 O_1 \quad \text{equation 3}$$

where: $C_1 = (\Delta t / K) + 2 X) / C_0$
 $C_2 = (\Delta t / K) - 2 X) / C_0$
 $C_3 = (2 (1 - X) - \Delta t / K) / C_0$
 $C_0 = \Delta t / K + 2 (1 - X)$.

C0, C1, C2, and C3 are dimensionless parameters.

An approximation for K is the travel time through the reach (length of reach divided by the average flow velocity). The value of X is between 0.0 and 0.5. A value of 0.0 gives maximum attenuation from the procedure and 0.5 provides the minimum attenuation. Linsley, Kohler, and Paulhus (1982) describe a procedure to determine K and X from hydrographs.

MUSKINGUM-CUNGE METHOD

Cunge (1969) developed equations to estimate K and X from hydraulic properties of the reach. The mathematical derivation is condensed and presented by Ponce (1981). The equations for K and X are:

$$X = \frac{1}{2} \left(1 - \frac{Q}{B S_0 c \Delta x} \right) \quad \text{equation 4}$$

where: c = flood wave celerity, ft/s
 Δx = distance increment, ft
 X = weighting factor, non-dimensional.
 B = bottom width or average width, ft.

and $K = \Delta x / c$.

DEFINITION AND SIGNIFICANCE OF RATING CURVE EXPONENT

The discharge versus area relationship for cross sections (rectangular, triangular, trapezoidal) may be fit by a power curve function of the form

$$Q = x A^m \quad \text{equation 5}$$

where: Q = Discharge in cfs,
 A = area in square feet,
 x = coefficient,
and m = exponent.

The exponent m has a physical meaning which can add to the understanding of the Muskingum-Cunge routing method. According to hydraulic theory, the speed at which a flood wave travels downstream is called the celerity and is equal to the slope of the discharge-area curve at a given discharge. In equation form

$$c = dQ / dA \quad \text{equation 6}$$

where: c = celerity, ft/sec.

Differentiating equation 5 with respect to area results in

$$c = dQ / dA = x m A^{m-1} = m (x A^m) / A = m Q / A \quad \text{equation 7}$$

$$c = m V$$

where: V = average velocity, ft/sec.

The exponent m is therefore a factor relating average velocity and wave velocity (celerity). The value of m used in the routing has a major effect on the calculation of travel time of the hydrograph through a reach, K . The larger the value of m , the shorter the travel time. Therefore the selection of m can affect the timing of peaks in any reach and can affect the way in which tributary hydrographs add to the peak of the hydrograph on the main stream.

The value of m equal to $5/3$ may be derived from the Manning equation. For cross sections where the hydraulic radius can be approximated by the depth (wide channel with bottom width = B), the equation takes the form:

$$Q = x A^m \quad \text{equation 8}$$

where: $x = (1.49 s^{1/2}) / (n B^{2/3})$ and $m = 5/3$

The discussion above concerning celerity and the Manning equation dealt with simple cross section shapes but is also applicable to most natural channel cross sections. This concept does not apply directly to cross sections with channel and flood plain. Discharge-area plots for these types of cross sections generally exhibit changes in slope and attempting to fit one power curve to the data can result in significant differences between the two curves.

If the slope of the discharge-area curve (plotted on log-log paper) is greater than one then the average velocity at the cross section is increasing with increasing discharge. If the slope should happen to be less than one, then average velocity is decreasing with increasing discharge. The third possibility is that the slope equals one and the velocity is constant regardless of discharge.

In a flood plain cross section Manning n is generally variable laterally (the Manning n in the channel is different from that in the flood plain). The word “segment” in the following discussion refers to a portion of a cross section with one assigned value of Manning n. For any single segment of a multi-segment cross section, the average velocity should increase with increasing discharge. However, when adding the discharge and area for all segments to get the total discharge and total area at a cross section there can be elevations where the average velocity actually decreases as flow increases. The most common occurrence of this is at discharges exceeding channel capacity with shallow depth in the flood plain for which the velocity in the channel is much larger than the velocity in the flood plain.

The slope of the discharge-area curve where the average velocity decreases (which can lead to a celerity significantly less than one) does not represent the true celerity or wave propagation speed. For this reason, a procedure to calculate m at a cross section was developed.

CALCULATION OF M FROM RATING TABLE DATA

The TR-20 program requires entry of a rating table to represent flow in a reach. Data required include elevation, discharge, area, top width, and friction slope for a minimum of three discharges (two greater than zero). When a rating is entered, m is computed at each discharge of the table. This procedure will reflect changes in slope of the discharge versus area curve typically occurring above the bankfull discharge and at large flood plain discharges.

The equations used to compute m are

$$m = S(2,3) \quad \text{for } Q < Q(3)$$

$$m(I) = \frac{Q(3) S(2,3) + \dots + (Q(I)-Q(I-1)) S(I-1,I)}{Q(I)} \quad \text{equation 9}$$

$$\text{for } Q(3) < Q < Q(I)$$

where: $m(I) = m$ at flow number I
 Q = discharge, cfs
 $Q(I)$ = discharge at subscript point number,
 $Q(1) = 0$
 $S(2,3)$ = log-log slope of discharge-area curve between points 2 and 3
 $S(I-1, I)$ = log-log slope of discharge-area curve between points $I-1$ and I .

m calculated this way can be considered a weighted slope. When m is desired at a non-tabulated point, m is linearly interpolated on a log-log basis.

If the log-log slope between each two consecutive points in a rating table is the same then m will be the same for all discharges, in other words, one power curve will fit the data accurately. When the log-log slope of the discharge-area curve changes then m will change. An example of calculation of m for a rating curve follows.

<u>Discharge, cfs</u>	<u>area sq. ft.</u>
0	0
200	110
500	200
870	800
1500	1200

Channel capacity is 500 cfs. At discharges higher than 500 cfs, the average velocity reduces due to low flood plain velocity. At discharges above 870 cfs, the average velocity increases again with deeper flow in the flood plain. The slope (log-log) between each pair of consecutive points is calculated:

$$S(2,3) = \log (500/200) / \log (200/110) = 1.5$$

$$S(3,4) = \log (870/500) / \log (800/200) = 0.4$$

$$S(4,5) = \log (1500/870) / \log (1200/800) = 1.3$$

For all discharges at or below 500 cfs, $m = 1.5$.

At a discharge of 870 cfs,

$$m = \frac{(500 \times 1.5) + ((870 - 500) \times 0.4)}{870} = 1.03$$

At a discharge of 1500 cfs,

$$m = \frac{(500 \times 1.5) + ((870 - 500) \times 0.4) + ((1500-870) \times 1.3)}{1500}$$

$m = 1.15$

At a non-tabulated discharge such as 1000 cfs, the value of m is interpolated between values of m at the higher and lower tabulated discharges.

SELECTION OF ROUTING TIME AND DISTANCE STEPS

Time and distance steps (Δt and Δx) are of critical importance in channel routing procedures. They are related in the Muskingum-Cunge routing procedure (the selection of Δt impacts the selection of Δx and vice versa). This is also true for the solution of the dynamic wave equations.

Δt . The time step used in the routing needs to represent the inflow hydrograph shape. In TR-20, the inflow hydrograph is determined from upstream land areas, reaches, reservoirs, etc. These are at a Δt which in most cases defines the hydrograph shape adequately. TR-20 uses the time increment of the inflow hydrograph as the Δt for the reach routing. However, if there are less than ten time intervals to the peak of the inflow hydrograph, the hydrograph is interpolated to provide ten intervals. The routing is completed at this shorter time increment. After the routing is completed, the hydrograph is interpolated back to its original time increment.

Δx . Selection of Δx depends on several factors. The reach length is determined by the user when the watershed is divided into subwatersheds. Obviously, smaller subwatersheds lead to shorter reach lengths and larger subwatersheds lead to longer reach lengths. This is not as critical a problem with the Muskingum-Cunge method as it was with the Modified Att-Kin (directly) or Convex methods (indirectly). Studies by Ponce documented in the 1981 report showed consistency of the Muskingum-Cunge (constant coefficient) method to develop very similar (not exact) outflow hydrographs for various selections of Δt and Δx . This characteristic of the Muskingum-Cunge is qualified in that the inflow hydrograph still needs to be defined consistently and that Δx not be significantly smaller than the distance traveled by the flood wave during a single time step. In other words, within a reasonable range of Δt and Δx , the Muskingum-Cunge produces consistent results. This is because the Muskingum-Cunge method selects Δx based on the desired Δt .

If reach lengths are rather short, in order for the routing to be reasonable, the time step should be short enough that the hydrograph peak is delayed at least one time step. If not, then the hydrograph will not have changed and there will be no impact of the short reach in TR-20. A number of such consecutive reaches may sum to a significant distance yet show no delay or attenuation of the hydrograph.

The Δx used in the Muskingum-Cunge method is based on Δt , wave celerity, reference discharge, top width, and reference slope. If the reach length is shorter than Δx , the reach is treated as a single step. If the Δx is less than $2/3$ of the reach length, the reach is divided into steps. For example if there are two steps, the routing of the reach is treated as if it were two separate routing reaches. The routing equations are solved for the upstream half of the reach and the resulting outflow hydrograph is routed through the downstream half of the reach.

DESCRIPTION OF TESTS

In preparing data for FLDWAV (Fread and Lewis, 1998) and TR-20, special care was taken to insure that geometric, hydrograph, and flow characteristics were consistently used.

TR-20 Muskingum-Cunge (constant coefficient): A total of 77 tests were completed using the National Weather Service FLDWAV model (dynamic wave equations) and TR-20 with Muskingum-Cunge routing. An inflow hydrograph was routed through a prismatic reach (cross section constant through the reach). In designing the tests, conditions were selected for which excellent comparisons were anticipated progressing to conditions which were expected to test the limits of the Muskingum procedure yet be in the realm of situations which could be encountered in NRCS hydrologic analysis. A total of 77 tests were conducted on 12 cross sections. At each cross section, one or more factors were varied. These factors included depth of water in the cross section (peak discharge of inflow hydrograph), reach length, slope, and time to peak of the inflow hydrograph. The HEC-RAS program was used to develop the steady flow rating table for use in TR-20. Each cross section was analyzed with downstream boundary condition of normal flow at the desired slope. The reference discharges tested in TR-20 were a) the average of peak inflow and base flow and b) peak inflow. Specific cross sections tested are listed below.

1. Rectangular channel with bottom width of 50 feet, slopes of 0.001 and 0.0005, length of 2 and 3 miles, peak discharge of 2000 cfs, base flow of 50 cfs, time to peak of 0.5 and 1.0 hour, and Manning n of 0.04.
2. Trapezoidal channel with bottom width of 30 feet, side slopes of 2:1, slopes of 0.001 and 0.005, length of 3 miles, peak discharge of 2000 cfs, base flow of 50 cfs, time to peak of 0.5 and 1.0 hour, and Manning n of 0.04.
3. Hypothetical cross section with channel and wide flat flood plain. The peak of the five different inflow hydrographs varied from 800 cfs (within the channel) to 10,000 cfs (deep flow in the flood plain). Results at reach lengths of 2, 4, 6, and 8 miles were obtained. Time to peak was 3.5 hours.
4. Cross section defined in the HEC-1 User's Manual figure 12.12, page 269. This is a cross section defined by 8 points. The peak discharges of the inflow hydrograph were 1600 cfs (within the channel), 4200 cfs (shallow flood plain flow), and 9000 cfs (deep flood plain flow). A reach length of 6 miles was used. The time to peak of the inflow hydrograph was 3.5 hours.
5. McPherson Creek, North Carolina, cross section 5. Peak of the inflow hydrographs were 500 and 4000 cfs. Reach length was 1.0 mile. Times to peak were 0.5, 1.0, and 1.5 hour. Slope was 0.0035.
6. Bayou Bourbeux, Louisiana, seven different cross sections. Four different peak inflows and two times to peak were tested. Reach length was 8000 feet, slopes ranged from 0.0003 to 0.001, and time to peak was 2 or 3.9 hours.

The seven cross sections from Bayou Bourbeux were selected to represent the range of cross section shapes including relative size of channel with respect to the flood plain. The relatively flat slopes in that watershed were considered a rigorous test of the Muskingum-Cunge routing method.

TR-20 Muskingum-Cunge (four-point variable coefficient): The four-point variable coefficient solution technique was run for Test Groups 1, 2, 3, 4, and one cross section from Bayou Bourbeaux (35 test runs).

SUMMARY OF FLOOD ROUTING TESTS

1. For TR-20 with Muskingum-Cunge (constant coefficient), having the reference flow equal to the peak flow is far superior to having it equal to the average of peak flow and base flow. Both peak discharge and time to peak of the outflow hydrograph are closer to the dynamic routing solution. This can be attributed to the weighted **m** routine which is used to estimate the flood wave celerity (used in estimation of Δx and routing coefficients). When the peak inflow is used as the reference discharge, the **m** is based on the rating curve from the channel bottom up to the peak flow. When the average of peak and base flow is used as the reference discharge, only the rating table from channel bottom up to this average flow is used. Information about discharges between the average flow and peak flow is not used. The constant coefficient routing always preserves the hydrograph volume.
2. For TR-20 with variable coefficient solution technique the volume of inflow did not equal the volume of outflow. These errors were generally less than 5 percent but 7 errors of more than 10 percent occurred. The peak discharge at the end of reach and its time of occurrence showed no improvement over the constant coefficient formulation.
3. The NWS FLDWAV model is one-dimensional with respect to flow direction (the water surface at a cross section is always horizontal). Muskingum-Cunge models are also one-dimensional and cannot be more accurate than the dynamic wave model because acceleration terms are ignored. Improvements could be made in the way in which the accuracy of the Muskingum-Cunge model is assessed by obtaining laboratory flume routing records, detailed natural channel and flood plain topography, stage versus time readings, and velocity measurements or apply a two-dimensional dynamic wave model.
4. A range of cross sections, reach lengths, slopes, and hydrograph rise times were tested. Most of the tests were rather harsh because of the amounts of attenuation involved. The tests need to be rigorous for several reasons. All simplified routing models do well for attenuations up to about 5 percent. Beyond that, there is increasing divergence between the dynamic wave routing and any simplified routing. Testing these non-stringent conditions is unproductive. The reason for adopting the Muskingum-Cunge method is to extend the applicability to a wider range of conditions. These application limits need to be determined.
5. Several analyses of results were attempted. The three most meaningful were A) separating the 77 tests into two groups representing channel flow (25 tests) and flood plain flow (52 tests), B) comparison of attenuation for the TR-20 Muskingum-Cunge (constant coefficient) and the dynamic wave routing, and C) comparing the error in the peak outflow with a non-dimensional factor representing the hydrograph rise time.
 - A) Separating the tests into channel and flood plain flow groups provided some interesting results. The group of channel tests included the rectangular and trapezoidal cross sections plus those flood plain cross sections where the flow was contained within the channel. The plot of attenuation of the 25 channel flow tests versus the dynamic wave routing produced a relationship with correlation coefficient (r squared) of 0.96. The best fit line through the results had a slight negative bias meaning that the Muskingum-Cunge

(constant coefficient) peak discharges were slightly lower than the dynamic routing. The plot of attenuation for the 52 flood plain flow tests versus the dynamic routing produced a correlation coefficient (r squared) of 0.70. The best-fit line through these results had a slight positive bias meaning that the Muskingum-Cunge (constant coefficient) peak discharges were on average slightly higher than the dynamic wave routing. This result indicates a significant difference in accuracy between these two groups. The Δx and routing coefficients for the channel routings are evidently more accurate than those for the flood plain routings. This also indicates that there is room for improvements in the way Δx and routing coefficients are determined for flood plain cross sections. More development is needed to determine what these improvements are. Some key items to investigate are the wave celerity and top width.

B) The attenuations for the Muskingum-Cunge (constant coefficient) and dynamic wave routings were plotted. The value of Q^* which represents attenuation in the following plot is defined as

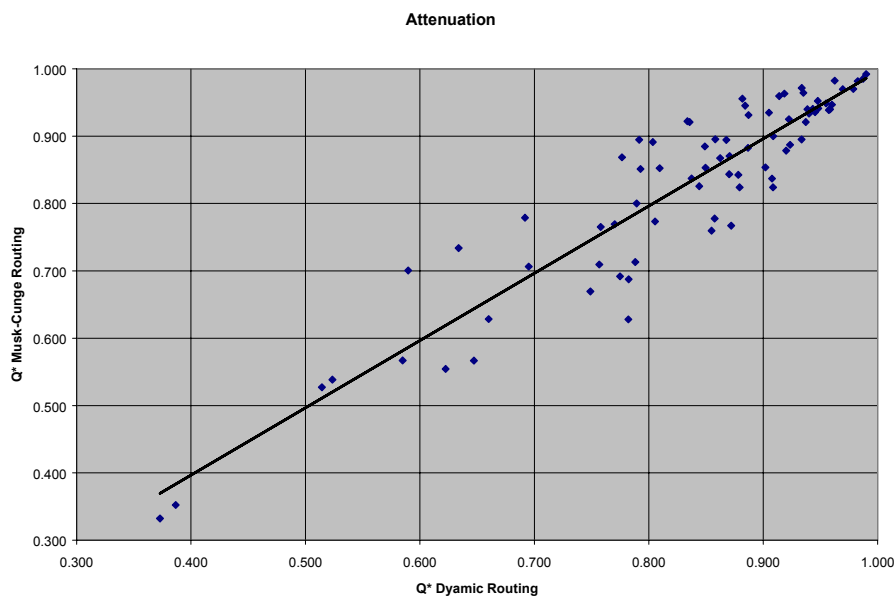
$$Q^* = (Q_o - Q_b) / (Q_p - Q_b)$$

Where Q_o = peak of outflow hydrograph, cfs

Q_p = peak of inflow hydrograph, cfs

Q_b = base flow, cfs

The entire group of tests had a correlation coefficient of 0.86 and a trend line which had a slope of 1:1. In other words, the best-fit line through the results showed no high or low bias of the results. For attenuations of 5 percent or less there was excellent agreement. Beyond 5 percent attenuation, results began to scatter some, however there were no tests with more than 20 percent error between the Muskingum-Cunge (constant coefficient) and dynamic routing peaks. Three tests had errors of 15 to 19 percent and fourteen tests had errors of 10 to 14 percent. That left 60 of the 77 tests with less than 10 percent error in peak at the end of the reach.



- C) With respect to limits on accuracy of the Muskingum-Cunge (constant coefficient) routing, promising results were obtained by displaying error in peak outflow versus a non-dimensional factor TQP*. This is defined as:

$$TQP^* = T_p Q_o S_o / A_o^{3/2} \quad \text{equation 10}$$

where : T_p = time to peak of the inflow hydrograph, seconds
 Q_o = peak discharge of the inflow hydrograph, cfs
 S_o = bed slope or friction slope if different, ft/ft
 A_o = cross section area corresponding to the discharge Q_o , square feet.

The values of Q_o , S_o , and A_o used in the equation for TQP* are based on the steady flow assumption, such as solving the Manning equation at a cross section. Small values of TQP* correspond to short rise times, very mild slopes, and low velocities. These are conditions where the disregard of the acceleration terms of the dynamic wave equations cause hydrologic routing methods to lose accuracy. As TQP* was less than 1.0, and approached zero, the error in the peak outflow increased. The value is easy to compute though some interpretations would have to be made. The main item needing interpretation is the time to peak of the inflow hydrograph, T_p . In the equation for TQP*, the value T_p is actually the rise time from base flow to peak flow. In a program like TR-20, there may be significant time at the beginning of the hydrograph where there is no or very little flow (such as the time when runoff begins).

6. The error in outflow peak time was computed based on the travel time of the peak discharge. The error is the time to peak outflow (Muskingum-Cunge) minus the time to the peak outflow (dynamic wave) divided by the time to peak outflow minus the time to the peak inflow of the dynamic wave routing. For the 77 tests, the average error in this timing for the Muskingum-Cunge (constant coefficient) was 6.8 percent with most values (60) ranging from plus to minus 20 percent. Errors in timing seemed to be random and not specifically related to any cross section or flow conditions. The sometimes large percent error in timing is deceptive because of the impact of Δt . For example, if the dynamic wave routing showed a travel time of 2 time steps and the Muskingum-Cunge (constant coefficient) showed a travel time of 3 time steps, the error in timing would be 50 percent. One could argue, in this case, that the time step should be reduced to get a better estimate of timing error. In customary practice, the largest time interval possible is used (to meet certain requirements to define hydrographs and reach travel times). To remain practical, in all of the tests, the Δt selected was 0.05 to 0.1 of the time to inflow hydrograph peak. Also, the criteria for time and distance step selection stated earlier was followed. The minimum travel time in any routing step (and this included the solution of the dynamic wave equations) was one time step. Of the 77 tests run, 48 had a timing error of one time step or less. 19 had a timing error of two time steps. This left 10 tests with a timing error of more than two time steps. The conclusion is that timing of routings is adequate.

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